Math 10A with Professor Stankova
Quiz 10; Wednesday, 11/1/2017
Section #107; Time: 11 AM
GSI name: Roy Zhao **Name:** _____

Circle True or False or leave blank. (1 point for correct answer, -1 for incorrect answer, 0 if left blank)

1. True **FALSE** The CDF is the derivative of the PDF.

> **Solution:** It is the opposite way around, the CDF is an antiderivative of the PDF.

2. True **FALSE** Suppose that $f(x) = x$ for $-0.5 \leq x \leq 1.5$ and 0 everywhere else. Since $\int_{-0.5}^{1.5} x dx = 1$ (you can assume the integral is correct), then f is a PDF.

> **Solution:** This is false since $f(-0.5) = -0.5$ which is negative and PDFs cannot be negative.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (5 points) Suppose that $f(x) = Cxe^{-x^2}$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$ for some constant C. If f is a PDF, then find C.

> **Solution:** Since f is a PDF, we require that
>
> $$\int_{-\infty}^{\infty} f(x)dx = \int_{0}^{\infty} Cxe^{-x^2}dx = 1.$$
>
> We u sub with $u = x^2$ and $du = 2xdx$ to get
>
> $$= \lim_{t\to\infty} \int_0^{t^2} Ce^{-u}/2du = \lim_{t\to\infty} -Ce^{-u}/2|_0^{t^2} = C/2.$$
>
> Therefore $C = 2$.

(b) (2 points) Find the CDF of f from above. (Hint: the CDF will be piecewise)

> **Solution:** For $x \leq 0$, then the CDF is 0 because the PDF is 0 there. Then for $x \geq 0$, we have that the CDF is
>
> $$F(x) = \int_{-\infty}^{x} f(t)dt = \int_0^x f(t)dt = -e^{-u}|_0^{x^2} = 1 - e^{-x^2}.$$

(c) (3 points) Find the probability that a randomly picked value from the PDF f from above is between 1 and 2.

Solution: The probability is

$$F(2) - F(1) = (1 - e^{-2^2}) - (1 - e^{-1^2}) = e^{-1} - e^{-4}.$$